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Bounds for Bose-Einstein correlation functions

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Abstract

Bounds for the correlation functions of identical bosons are discussed for the general case of a Gaussian density matrix. In particular, for a purely chaotic system the two-particle correlation function must always be greater than one. On the other hand, in the presence of a coherent component the correlation function may take values below unity. The experimental situation is briefly discussed.

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The most important method to obtain information about space-time aspects of particle production in high energy collisions is particle interferometry. One studies the correlation function of m ($m \geq 2$) identical particles,

$$C_m(\vec{k}_1, ..., \vec{k}_m) \;=\; \frac{P_m(\vec{k}_1, ..., \vec{k}_m)}{P_1(\vec{k}_1) \cdot ... \cdot P_1(\vec{k}_m)} \tag{1}$$

where $\vec{k}_i$ is the three-momentum of the i-th particle and $P_\ell(\vec{k}_1, ..., \vec{k}_\ell)$ are the ℓ-particle single inclusive distributions. For ultrarelativistic collisions, one usually considers bosons, i.e., one measures Bose-Einstein correlations (BEC).

In practice most of the experimental measurements have been restricted so far to two particle correlations (see, however, ref. [1]). Moreover, with very few exceptions, only identical charged particles were considered. Usually the data are analysed by fitting them to a form

$$C_2(\vec{k}_1, \vec{k}_2) \;=\; 1 \;+\; |\tilde{\rho}(k_1 - k_2)|^2 \tag{2}$$

where $\tilde{\rho}(q)$ is the Fourier transform of the space-time distribution of the source elements,

$$\tilde{\rho}(q) \;=\; \int d^4x \; \rho(x) \; \exp(iq_\mu x^\mu) \tag{3}$$

Thus, the inverse widths of the correlation function are related to the size and lifetime of the source. Eq. (2) was first derived by symmetrizing the two particle wave function and averaging over the emission points of the particles *after* taking the square of the matrix element [2]. This corresponds to the assumption of a purely chaotic source (random phases). From Eq. (2) one obtains

$$1 \;\leq\; C_2(\vec{k}_1, \vec{k}_2) \;\leq\; 2 \tag{4}$$

and these are the bounds for C_2 in the conventional treatment of BEC. For neutral particles as well as for a partially coherent source, modifications of these bounds appear (cf. below). The existence of bounds for BEC is important, among other things, because they can serve as checks for results derived in any particular approach.

A more general approach to BEC is based on the density matrix formalism, i.e., a formalism which does not assume pure states (wave functions). To be specific, we shall consider the case (which covers almost all phenomena known in quantum statistics) that the multiparticle system is described by a *Gaussian density matrix* [1]. If the particles are emitted from a large number of independent source elements, such a form of the density matrix follows from the central limit theorem (cf., e.g., [3]). Upper bounds for the two-particle correlation function for a Gaussian density matrix have already been discussed in [4]: for charged bosons, one finds again $C_2(\vec{k}_1, \vec{k}_2) \leq 2$, but for some neutral bosons like π^0's and photons, $C_2(\vec{k}_1, \vec{k}_2) \leq 3$. As will be demonstrated below, concerning the lower bounds one can derive the following general results: (a) for a purely chaotic source, $C_2(\vec{k}_1, \vec{k}_2) \geq 1$, and (b) for a

[1] We do not treat final state interactions here.

partially coherent source, $C_2(\vec{k}_1, \vec{k}_2) \geq 2/3$ for identical charged bosons, while for π^0's and photons, $C_2(\vec{k}_1, \vec{k}_2) \geq 1/3$.

For a Gaussian density matrix, all multiparticle inclusive distributions can be expressed [4] in terms of the quantities

$$D(k_r, k_s) \equiv \sqrt{E_r E_s}\ \langle a^\dagger(\vec{k}_r) a(\vec{k}_s) \rangle \tag{5}$$

$$\tilde{D}(k_r, k_s) \equiv -\sqrt{E_r E_s}\ \langle a(\vec{k}_r) a(\vec{k}_s) \rangle \tag{6}$$

$$I(k_r) \equiv -i \sqrt{E_r}\ \langle a(\vec{k}_r) \rangle \tag{7}$$

where $a^\dagger(\vec{k})$ and $a(\vec{k})$ are the creation and annihilation operators of a particle of momentum $\vec{k}$, and the indices r, s label the particles.

For the general case of a partially coherent source, the single inclusive distribution can be expressed as the sum of a chaotic component and a coherent component,

$$P_1(\vec{k}) \ = \ P_1^{chao}(\vec{k}) \ + \ P_1^{coh}(\vec{k}) \tag{8}$$

with

$$P_1^{chao}(\vec{k}) \ = \ D(k, k) \tag{9}$$

and

$$P_1^{coh}(\vec{k}) \ = \ |I(k)|^2 \tag{10}$$

The chaoticity parameter p, representing the ratio between the mean number of chaotically produced particles and the mean total multiplicity, is then

$$p(k) \ = \ \frac{D(k, k)}{D(k, k) + |I(k)|^2} \tag{11}$$

To write down the correlation functions in a concise form, it is useful to introduce the normalized current correlators,

$$d_{rs} = \frac{D(k_r, k_s)}{[D(k_r, k_r) \cdot D(k_s, k_s)]^{\frac{1}{2}}}, \quad \tilde{d}_{rs} = \frac{\tilde{D}(k_r, k_s)}{[D(k_r, k_r) \cdot D(k_s, k_s)]^{\frac{1}{2}}}, \tag{12}$$

where the indices r, s label the particles. Since $d(k, k')$ and $\tilde{d}(k, k')$ are in in general complex valued, one may prefer to express the correlation functions in terms of the magnitudes and the phases,

$$\begin{aligned} T_{rs} \equiv T(k_r, k_s) \ &= \ |d(k_r, k_s)| \\ \tilde{T}_{rs} \equiv \tilde{T}(k_r, k_s) \ &= \ |\tilde{d}(k_r, k_s)| \\ \phi_{rs}^{ch} \equiv \phi^{ch}(k_r, k_s) \ &= \ \mathrm{Arg}\ d(k_r, k_s) \\ \tilde{\phi}_{rs}^{ch} \equiv \tilde{\phi}^{ch}(k_r, k_s) \ &= \ \mathrm{Arg}\ \tilde{d}(k_r, k_s) \end{aligned} \tag{13}$$

and the phase of the coherent component,

$$\phi_r^c \;\equiv\; \phi^c(k_r) \;=\; \text{Arg}\; I(k_r)\,. \tag{14}$$

The same notation will be employed for the chaoticity parameter,

$$p_r \;\equiv\; p(k_r) \tag{15}$$

For identical charged bosons (e.g., π^-) the two-particle correlation function reads

$$C_2^{--}(\vec{k}_1,\vec{k}_2) \;=\; 1+\; 2\sqrt{p_1(1-p_1)\cdot p_2(1-p_2)}\; T_{12}\;\cos(\phi_{12}^{ch}-\phi_1^c+\phi_2^c) \;+\; p_1p_2\; T_{12}^2 \tag{16}$$

For neutral bosons like photons, or π^0's, the terms $\tilde{d}(k_r,k_s)$ may contribute[2] to the BEC function[4]:

$$\begin{aligned}C_2^{00}(\vec{k}_1,\vec{k}_2)=&1+2\sqrt{p_1(1-p_1)\cdot p_2(1-p_2)}\; T_{12}\;\cos(\phi_{12}^{ch}-\phi_1^c+\phi_2^c) \;+\; p_1p_2\; T_{12}^2\\ &+2\sqrt{p_1(1-p_1)\cdot p_2(1-p_2)}\; \tilde{T}_{12}\;\cos(\tilde{\phi}_{12}^{ch}-\phi_1^c-\phi_2^c) \;+\; p_1p_2\; \tilde{T}_{12}^2\end{aligned} \tag{17}$$

Let us first consider the case of a purely chaotic source. Insertion of $p(k)\equiv 1$ in Eqs. (16) and (17) immediately yields $C_2(\vec{k}_1,\vec{k}_2)\geq 1$. In the case of partial coherence, the terms containing cosines come into play and consequently C_2 may take values below unity. Eqs. (16,17) imply that $C_2^{--}(\vec{k}_1,\vec{k}_2)\geq 2/3$ and $C_2^{00}(\vec{k}_1,\vec{k}_2)\geq 1/3$. Because of the cosine functions in (16,17) one would expect C_2 as a function of the momentum difference q to oscillate between values above and below 1. Indeed such a behaviour of the Bose-Einstein correlation function has been observed in high energy e^+e^- collision experiments (cf., e.g., ref. [5]), but apparently not in hadronic reactions. This observation was interpreted as a consequence of final state interactions in ref. [6]. If final state interactions determine this effect, it is unclear why the effect is not seen in hadronic reactions. On the other hand, if coherence is responsible for it, this would be easier to understand. Indeed multiplicity distributions of secondaries in e^+e^- reactions are much narrower (almost Poisson-like) than in pp reactions, which is consistent with the statement that hadronic reactions are more chaotic than e^+e^- reactions [7].

So far, two methods have been proposed for the detection of coherence in BEC: the intercept criterion [8] ($C_2(\vec{k},\vec{k})<2$) and the two exponent structure of C_2 [9]. Both these methods have their difficulties because of statistics problems and other competing effects. The observation of $C_2(\vec{k}_1,\vec{k}_2)<1$ could constitute a third criterion for coherence, although $C_2(\vec{k}_1,\vec{k}_2)<1$ is not a necessary condition.

[2]These terms are expected to play a significant role only for soft particles of energies of the order of the inverse lifetime of the source, or for sources with very small lifetimes [4].

Recently, the two-particle correlation function has been calculated for photons emitted from a longitudinally expanding system of hot and dense hadronic matter created in ultra-relativistic nuclear collisions [10]. For such a system, the particles are emitted from a large number of independent source elements (fluid elements), and consequently, one would expect the multiparticle final state to be described by a Gaussian density matrix. However, although the system is assumed to be purely chaotic (i.e., it does not contain a coherent component) the correlation function calculated in [10] is found to take values significantly below unity. Clearly, this is in in contradiction with the general result derived above from quantum statistics ($C_2 \geq 1$ for a chaotic system).

The expression for the two-particle inclusive distribution used in ref. [10] (equation (3) of that paper), which in our notation takes the form ($w(x,k)$ is the emission rate)

$$P_2(\vec{k}_1, \vec{k}_2) \;=\; \int d^4x_1 \int d^4x_2 \; w\,(x_1, k_1)\;\; w\,(x_2, k_2)\;\; [1 \;+\; \cos((k_1 - k_2)(x_1 - x_2))] \tag{18}$$

does not exclude values below unity for the two-particle correlation function[3]. Thus, the fact that the authors of ref. [10] find values below one for C_2 may be due to the application of an inadequate expression for $P_2(\vec{k}_1, \vec{k}_2)$ [4] [5].

Below, we shall briefly derive expressions for $P_2(\vec{k}_1, \vec{k}_2)$ which could be used for the problem treated in [10, 11] and which do not suffer from the deficiencies mentioned above. To this end, we consider two conventional approaches used in this field: (i) the classical current formalism, and (ii) symmetrization of the wave function.

(i) In the current formalism particle sources are described in terms of a distribution of classical currents [12, 13, 14]. For a Gaussian distribution of currents and in the absence

[3] To see this, consider, e.g., the simple ansatz

$$w(x,k) \;=\; const.\; \exp[-\alpha(\vec{x} - \beta\vec{k})^2]\; \delta(t - t_0)$$

where α and β are free parameters. The expression for $P_2(\vec{k}_1, \vec{k}_2)$ used in ref.[10] (cf. eq. (18)) then yields

$$C_2(\vec{k}_1, \vec{k}_2) \;=\; 1 \;+\; \exp\left[-\frac{\vec{q}^{\,2}}{2\alpha}\right]\; \cos[\beta\vec{q}^2]$$

Clearly, if β exceeds α^{-1} the above expression will oscillate and take values below unity. On the other hand, in the current formalism (cf. below) one obtains with the same ansatz for w

$$C_2(\vec{k}_1, \vec{k}_2) \;=\; 1 \;+\; \exp\left[-\frac{\vec{q}^{\,2}}{2\alpha}\right] \;\geq\; 1.$$

[4] Another possibility which was pointed out by the referee of the present paper is that the values below one obtained for C_2 in the calculations of ref. [10] are caused by a certain approximation used to evaluate the integrals in [10].

[5] In a recent publication [11] the authors of ref. [10] have included the effects of transverse expansion. The resultant correlation functions do not take values below one. However, if the results for the 1-dimensional expansion are affected by the use of an inadequate expression for $P_2(\vec{k}_1, \vec{k}_2)$, this would also cast doubts on the results obtained for the 3-dimensionally expanding system.

of a coherent component, all multiparticle distributions can be expressed in terms of the two-current correlator, $\langle J^\star(x)J(x')\rangle$ [4]. The one- and two-particle inclusive distributions then take the form[6]

$$P_1(\vec{k}) \;=\; \int d^4x \; w(x,k) \tag{19}$$

and

$$P_2(\vec{k}_1,\vec{k}_2) \;=\; \int d^4x_1 \int d^4x_2 \left[w\,(x_1,k_1)\, w\,(x_2,k_2) \;+\; w\,(x_1,K)\, w\,(x_2,K)\;\; \exp\left[iq_\mu(x_1^\mu - x_2^\mu)\right]\right] \tag{20}$$

where $K^\mu = (k_1^\mu + k_2^\mu)/2$ and $q^\mu = k_1^\mu - k_2^\mu$ are the mean momentum and momentum difference of the pair, and where

$$w(x,k) \;=\; \int d^4y \; \left\langle J^\star(x+\frac{y}{2})J(x-\frac{y}{2})\right\rangle \;\exp\left[-ik_\mu y^\mu\right] \tag{21}$$

(ii) In the wave function approach, the amplitude for emission of two identical particles of momenta k_1, k_2 from the space-time points x_1, x_2 is

$$\begin{aligned} A(k_1,k_2) =& M(k_1,x_1)M(k_2,x_2)\exp[i(k_1x_1+k_2x_2)] \\ &+ M(k_1,x_2)M(k_2,x_1)\exp[i(k_1x_2+k_2x_1)] \end{aligned} \tag{22}$$

The one- and two-particle inclusive distributions are

$$P_1(\vec{k}) \;=\; \int d^4x \; |M(k,x)|^2 \tag{23}$$

and

$$\begin{aligned} P_2(\vec{k}_1,\vec{k}_2) \equiv& \frac{1}{2}\int d^4x_1 \int d^4x_2 \;\; |A(k_1,k_2)|^2 \\ =& \int d^4x_1 \int d^4x_2 \quad \left[|M(k_1,x_1)|^2\, |M(k_2,x_2)|^2 \right. \\ &\left. +\;\; M(k_1,x_1)M^\star(k_2,x_1)M(k_2,x_2)M^\star(k_1,x_2)\;\; \exp\left[iq_\mu(x_1^\mu - x_2^\mu)\right]\right] \end{aligned} \tag{24}$$

Both expressions [7] (20) and (24) can be cast in the form

$$P_2(\vec{k}_1,\vec{k}_2) \;=\; P_1(\vec{k}_1)P_1(\vec{k}_2) \;+\; D(k_1,k_2)D^\star(k_1,k_2) \tag{25}$$

[6] For neutral particles, there may be an additional contribution to $P_2(\vec{k}_1,\vec{k}_2)$ which only plays a role for soft particles and which will be neglected here, cf. footnote 2 on page 4.

[7] To obtain specific results for $P_2(\vec{k}_1,\vec{k}_2)$ for photons emitted from an expanding quark-gluon-plasma as discussed in [10], it is reasonable to substitute the emission rates[15] for the integrands in Eqs. (19) and (23), i.e.,

$$w(x,k) = |M(k,x)|^2 = const.\; T^2 \ln\left(\frac{2.9\; k_\mu u^\mu}{g^2 T} \;+\; 1\right)\; \exp\left[-\frac{k_\mu u^\mu}{T}\right]$$

where u^μ is the four velocity of the fluid, T the temperature and g the QCD coupling constant.

where for the current formalism

$$D(k_1,k_2) \;=\; \int d^4x\; w\left(x, \frac{k_1+k_2}{2}\right)\; \exp[-ix_\mu(k_1^\mu - k_2^\mu)] \tag{26}$$

and for the wave function approach

$$D(k_1,k_2) \;=\; \int d^4x\; M^\star(k_1,x)M(k_2,x)\; \exp[-ix_\mu(k_1^\mu - k_2^\mu)] \tag{27}$$

It then follows from Eq. (25) that $C_2(\vec{k}_1,\vec{k}_2) \geq 1$ both for the results of (i) and of (ii).

The above considerations concerning bounds for the BEC functions refer to the case of a Gaussian density matrix. In general, a different form of the density matrix may yield correlation functions that are not constrained by the bounds derived here. For instance, for a system of squeezed states C_2 can take arbitrary positive values [16]. Moreover, for particles produced in high energy hadronic or nuclear collisions, the fluctuations of quantities such as impact parameter or inelasticity may introduce additional correlations which may destroy the Gaussian form of the density matrix and also affect the bounds of the BEC functions (cf. ref. [17], [18]).

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